ELSEVIER

Journal of Computational and Applied Mathematics 137 (2001) 1–12

JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

www.elsevier.com/locate/cam

A study of splitting scheme for hyperbolic conservation laws with source terms

L.A. Monthé *

LMI-INSA de Rouen, UPRES-A CNRS 6085, BP 08 Place Emile Blondel, 76130 Mont-Saint-Aignan Cedex, France

Received 5 June 1999; received in revised form 7 September 1999

Abstract

This study deals with the convergence of a numerical scheme for conservation laws including source terms. A splitting method for source term integration is presented. More precisely, the convergence of the numerical solution towards the entropy solution is proved in the scalar case. Because of the effect of source term, the constructed scheme is total variation bounded. Numerical experiments for one-dimensional shallow water equation are presented to demonstrate the performance of the scheme.

Keywords: Hyperbolic; Source terms; *BV* stability; Splitting method; Shallow water

1. Introduction

Many physical problems of fluid dynamics are nonhomogeneous and governed by hyperbolic conservation laws with nonvanishing source terms, one can refer, for example, in water flow to Macchione and Morelli [10], Monthé [11] and in combustion problem to Leveque and Yee [8] and Elmahi and Benkhaldoun [4]. The source term is related to physical effects (exterior forces, chemical reacting gas, etc.) or to geometrical effects (axisymmetric or cylindric problems, area with variable section, etc.).

A review of homogeneous case has been given recently in [12]. The theory of nonhomogeneous scalar may be found in [7], while for the system one can refer to Liu [9], who proved the global existence due to the asymptotic behaviour of the solution. Other theoretical results for particular nonhomogeneous hyperbolic systems which used the estimation of the Riemann invariants can be found in [17]. More recently, there have been some contributions for the approximation of conservation laws involving source terms in [3,5,14]. These numerical methods are based on explicit difference

* Tel.: +33-2-35-52-83-00; fax: +33-2-35-52-83-32.
E-mail address: monthe@lmi.insa-rouen.fr (L.A. Monthé).

0377-0427/01/$ - see front matter
PII: S0377-0427(00)00615-4

schemes. The solution of nonhomogeneous equations does not possess total variation diminishing (TVD) property because of the effect of the source term which increases the total variation. A numerical integration along the characteristic curves is proposed by Benkhaldoun and Chalabi [2]. It is well known that explicit schemes are not appropriate for the numerical treatment of the source terms in some case, this motivates the use of splitting schemes. Recently, the approximation of stiff case was studied by several authors [8,6,14].

In this paper, we study the convergence of the approximate solution obtained by splitting scheme where the hyperbolic part is approximated using a finite volume monotone scheme. In this study, we assume that the source term is Lipschitzian. Owing to the implicit character for taking into account source term, the proposed scheme is TVB, and entropy satisfying at the limit. This paper is structured as follows:

Section 2 is devoted to the motivation of the scheme on the linear scalar advection equation. In Section 3, we present some preliminaries related to the nonhomogeneous scalar conservation laws. Section 4 concerns the splitting scheme where the covergence of the numerical solution towards the entropy solution is obtained. Numerical experiments are presented in Section 5, in which the hyperbolic part is computed with a second-order-accurate scheme.

2. Motivation

In order to motivate the method we shall introduce in this paper, we consider the following linear scalar equation with source term:

$$\frac{\partial u}{\partial t} + a\,\frac{\partial u}{\partial x} = s(x,u), \tag{1}$$

where a is assumed to be a positive constant for simplicity. By using an upwind scheme to compute the convective part of (1), one has

$$u_j^{n+1} = u_j^n - \sigma(u_j^n - u_{j-1}^n) + \Delta t s_j^n, \tag{2}$$

where $\sigma = a\Delta t/\Delta x$ is the Courant number, supposed to be less than one. Several choices are possible to approximate s_j^n. In order to give sufficient conditions for the linear stability by the Fourier transform, and the linear positivity conditions of Eq. (2), we consider $s(x,u) = -vu$. In that case, three possibilities for s_j^n calculation are analyzed in what follows:

(i) $s_j^n = -vu_j^n$ is the centred discretization.
(ii) $s_j^n = -v(u_j^n + u_{j-1}^n)/2$ is the upwinded approximation.
(iii) Use of splitting method and impliciting the source operator we obtain

$$\frac{\tilde{u}_j - u_j^n}{\Delta t} = -v\tilde{u}_j,$$

$$u_j^{n+1} = \tilde{u}_j - \sigma(\tilde{u}_j - \tilde{u}_{j-1}).$$

Recall that positivity condition requires that all coefficients are positive in the linear scheme. The main results are represented in Table 1 and illustrated in Fig. 1, where $\delta = v\Delta t/2$.

One can note the advantage of using the splitting approach.

Table 1
Conditions for linear stability and positivity

	Centred	Upwinded	Splitting
Linear stability	$\sigma+\delta\leqslant 1$	$\sigma\leqslant 1$ $\delta\leqslant 1$	$\sigma\leqslant 1+\delta$
Positivity conditions	$\sigma+2\delta\leqslant 1$	$\delta\leqslant\sigma$ $\sigma+\delta\leqslant 1$	$\sigma\leqslant 1$

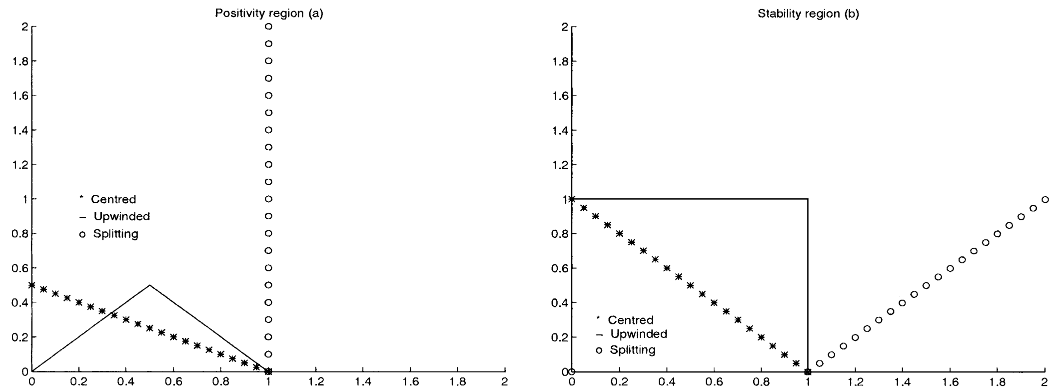

Fig. 1. Positivity region (a) and linear stability region (b) in $\sigma\delta$ ($\sigma = a\Delta t/\Delta x$, $\delta = v\Delta t/2$) plan.

3. Study of splitting scheme

3.1. Preliminaries

The nonhomogeneous scalar conservation law to be investigated, is represented in this section by the following Cauchy problem:

$$\frac{\partial u}{\partial t}+\frac{\partial f(u)}{\partial x}=q(u), \quad x\in\mathbb{R}, \quad T>t>0, \tag{3}$$

$$u(x,0)=u_0(x), \quad x\in\mathbb{R}. \tag{4}$$

Some nonrestrictive hypotheses are made on the data of the problem

$$u_0\in BV(\mathbb{R}), \tag{5}$$

$$q\in C^m(\mathbb{R}) \text{ is Lipschitz}, \quad f\in C^m(\mathbb{R}), \quad m\geqslant 2, \tag{6}$$

$$q(0)=0. \tag{7}$$

The conditions (5) and (6) are rather classical; we shall denote by k the Lipschitz constant of q; condition (7) is natural since in general there is neither production nor consumption when there is no mass or energy.

We seek a weak solution to the Cauchy problem (3)–(4), defined as a function $u \in L^\infty(\mathbb{R}\times]0,T[)$ satisfying

$$\int_{\mathbb{R}}\int_0^T u\,\frac{\partial\phi}{\partial t} + f(u)\,\frac{\partial\phi}{\partial x}\,\mathrm{d}x\,\mathrm{d}t + \int_{\mathbb{R}} u_0(x)\phi(x,0)\,\mathrm{d}x = -\int_{\mathbb{R}}\int_0^T q(u)\phi(x,t)\,\mathrm{d}x\,\mathrm{d}t, \tag{8}$$

for all test functions $\phi \in \mathscr{D}(\mathbb{R}\times[0,T[)$, with compact support in $\mathbb{R}\times[0,T[$. Since there are many weak solutions in general, one adds an entropy condition to select the physically correct one characterized by the following entropy condition:

$$\int_{\mathbb{R}}\int_0^T \eta(u)\,\frac{\partial\phi}{\partial t} + F(u)\,\frac{\partial\phi}{\partial x}\,\mathrm{d}x\,\mathrm{d}t \geqslant -\int_{\mathbb{R}}\int_0^T \eta'(u)q(u)\phi(x,t)\,\mathrm{d}x\,\mathrm{d}t,$$

$$\forall\phi \in \mathscr{D}(\mathbb{R}\times]0,T[),\ \phi\geqslant 0, \tag{9}$$

for all convex entropy $\eta \in C^2(\mathbb{R})$ and the associated entropy flux F satisfying

$$\eta'(u)f'(u) = F'(u). \tag{10}$$

Let us recall that the main theoretical result is due to Kruzkov [7]. We introduce some notations: $\tau \in \mathbb{R}$ is the time step, $h=\tau/r$ is the spatial size of the mesh, with $r \in \mathbb{R}$ kept constant; $t_n = n\tau,\ n \in \mathbb{N}$, $x_j = jh,\ j \in \mathbb{Z}$;

$$T_n = [t_n, t_{n+1}[, \quad I_j = [x_{j-1/2}, x_{j+1/2}[.$$

We consider the approximate solution u_τ of the weak solution of (3) and (4) with

$$u_\tau(x,t) = u_j^n \quad \text{for } (x,t) \in I_j \times T_n,$$

and the initial condition (4) is projected onto the space of piecewise constant functions as

$$u_j^0 = \frac{1}{h}\int_{I_j} u_0(x)\,\mathrm{d}x \quad \forall j \in \mathbb{Z}. \tag{11}$$

3.2. Stability and convergence

We use constant piecewise data to take into account the nonhomogeneous character in the numerical solution. The source term is handled by solving implicitly an ordinary differential equation and after then treating the hyperbolic part explicitly. We have

$$\frac{\overline{u}_j^n - u_j^n}{\tau} = q(\overline{u}_j^n), \tag{12}$$

$$u_j^{n+1} = \overline{u}_j^n - r[g(\overline{u}_j^n, \overline{u}_{j+1}^n) - g(\overline{u}_{j-1}^n, \overline{u}_j^n)], \tag{13}$$

where g is a local Lipschitz numerical flux of a three points conservative scheme. The explicit scheme is monotone under the following CFL condition:

$$r\max_{w,z}|g(u,w) - g(v,w)| + |g(z,u) - g(z,v)| \leqslant |u-v| \quad \forall u,v \in A, \tag{14}$$

where

$$A = \{v \in L^\infty(\mathbb{R}), \|v\|_{L^\infty(\mathbb{R})} \leqslant \gamma_0\|u_0\|_{L^\infty(\mathbb{R})}\} \tag{15}$$

and γ_0 is a strictly positive number.

Lemma 1. *Under the condition*

$$\tau k < 1 \tag{16}$$

the scheme (12) *verifies the following two inequalities*:

$$\|\bar{u}^{\,n}\|_{L^\infty(\mathbb{Z})} \leqslant \frac{1}{(1-\tau k)} \|u^n\|_{L^\infty(\mathbb{Z})}, \tag{17}$$

$$TV(\bar{u}^{\,n}) \leqslant \frac{1}{(1-\tau k)} TV(u^n). \tag{18}$$

Proof. Let θ be a function defined by $\theta(x) = x - \tau q(x)$. Under the condition (16), we have $\theta'(x) = 1 - \tau q'(x) \geqslant 1 - \tau k > 0$, θ is a strictly increasing function and Eq. (12) has a unique solution.

In addition, $|x| \leqslant |\theta(x)| + \tau|q(x)| \leqslant |\theta(x)| + \tau k|x|$, hence,

$$\|\bar{u}^{\,n}\|_{L^\infty(\mathbb{Z})} \leqslant \frac{1}{(1-\tau k)} \|u^n\|_{L^\infty(\mathbb{Z})}.$$

Using the same arguments

$$\bar{u}^n_{j+1} - \bar{u}^n_j = u^n_{j+1} - u^n_j + \tau[q(\bar{u}^n_{j+1}) - q(\bar{u}^n_j)]$$

thus

$$|\bar{u}^n_{j+1} - \bar{u}^n_j| \leqslant |u^n_{j+1} - u^n_j| + \tau k|\bar{u}^n_{j+1} - \bar{u}^n_j|$$

and that ends the proof of the lemma. □

Proposition 2. *If the* CFL *conditions* (14) *and* (16) *are satisfied*, *then schemes* (12), (13) *hold the following inequalities*:

$$\|u^{n+1}\|_{L^\infty(\mathbb{Z})} \leqslant \mathrm{e}^{\delta_0 T} \|u^0\|_{L^\infty(\mathbb{Z})}, \tag{19}$$

$$TV(u^{n+1}) \leqslant \mathrm{e}^{\delta_0 T} TV(u^0), \tag{20}$$

where δ_0 is a strictly positive constant.

Proof. Taking into account the CFL condition (14) and condition (16), we show easily that

$$\|u^{n+1}\|_{L^\infty(\mathbb{Z})} \leqslant \|\bar{u}^{\,n}\|_{L^\infty(\mathbb{Z})}$$

and

$$TV(u^{n+1}) \leqslant TV(\bar{u}^{\,n})$$

with the use of conditions (17) and (18), we have

$$\|u^{n+1}\|_{L^\infty(\mathbb{Z})} \leqslant \frac{1}{(1-\tau k)} \|u^n\|_{L^\infty(\mathbb{Z})}$$

and similarly

$$TV(u^{n+1}) \leqslant \frac{1}{(1-\tau k)} TV(u^n).$$

If we set $\delta = k/(1-\tau k)$, the above inequalities can be rewritten as

$$\|u^{n+1}\|_{L^\infty(\mathbb{Z})} \leqslant (1+\delta\tau)\|u^n\|_{L^\infty(\mathbb{Z})}$$

and

$$TV(u^{n+1}) \leqslant (1+\delta\tau) TV(u^n),$$

which implies

$$\|u^n\|_{L^\infty(\mathbb{Z})} \leqslant \mathrm{e}^{\delta n\tau}\|u^0\|_{L^\infty(\mathbb{Z})} \leqslant \mathrm{e}^{\delta T}\|u^0\|_{L^\infty(\mathbb{Z})}$$

and

$$TV(u^n) \leqslant \mathrm{e}^{\delta n\tau}\ TV(u^0) \leqslant \mathrm{e}^{\delta T} TV(u^0).$$

Moreover,

$$\exists \tau_0 > 0/\forall \tau < \tau_0, \quad \delta = \frac{k}{1-\tau k} < \delta_0 = \frac{k}{1-\tau_0 k},$$

then we obtain

$$\|u^n\| \leqslant \mathrm{e}^{\delta_0 T}\|u^0\|_{L^\infty(\mathbb{Z})}$$

and

$$TV(u^n) \leqslant \mathrm{e}^{\delta_0 T}\ TV(u^0). \quad \square$$

Remark 3. We only established the BV stability in space; this is sufficient for BV stability in space and time according to Chalabi [3].

Theorem 4. *If* $u_0 \in L^\infty(\mathbb{R}) \cap L^1(\mathbb{R})$, $f \in C^1(\mathbb{R})$, $q \in C^1(\mathbb{R})$, *such that* $q(0)=0$ *and* $|q'| \leqslant k$, *then under the* CFL *condition* (14) *and condition* (16), *the approximate solution* u_τ *constructed by the splitting scheme* (12) *and* (13) *converges in* $L^1_{\mathrm{loc}}(\mathbb{R}\times]0,T[)$ *towards the entropy solution satisfying* $(3,\ldots,7)$, *as* τ *tends to zero.*

Proof. From Proposition 1, the sequence (u_τ) is bounded in $L^\infty(\mathbb{R}\times]0,\ T[) \cap BV(\mathbb{R}\times]0,T[)$, then by Helly's theorem, we can extract a subsequence still labelled u_τ which converges towards u in $L^1_{\mathrm{loc}}(\mathbb{R}\times]0,\ T[)$. Elsewhere, we have

$$\|\bar{u}^{\,n}\|_{L^\infty(\mathbb{Z})} \leqslant \frac{1}{(1-\tau k)}\ \|u^n\|_{L^\infty(\mathbb{Z})} \leqslant \frac{1}{(1-\tau_0 k)}\ \mathrm{e}^{\delta_0 T}\|u^0\|_{L^\infty(\mathbb{Z})} \tag{21}$$

and similarly

$$TV(\bar{u}^{\,n}) \leqslant \frac{1}{(1-\tau_0 k)}\ \mathrm{e}^{\delta_0 T}\ TV(u_0). \tag{22}$$

Define

$$\bar{u}_\tau(x,t) = \bar{u}^n_j \quad \text{for } (x,t) \in I_j \times T_n,$$

and by the same arguments, we can extract a subsequence $\bar{u}_\tau$ which converges towards v in $L^1_{\mathrm{loc}}(\mathbb{R}\times]0,T[)$.

Now, we shall show that $u=v$ in $L^1_{\mathrm{loc}}(\mathbb{R}\times]0,T[)$. To do this, we consider a compact set $K\subset\mathbb{R}\times]0,T[$, then we have

$$\|u-v\|_{L^1(K)}\leqslant\|u-u_\tau\|_{L^1(K)}+\|u_\tau-\bar{u}_\tau\|_{L^1(K)}+\|\bar{u}_\tau-v\|_{L^1(K)}.$$

Since

$$|\bar{u}^n_j-u^n_j|=|\tau q(\bar{u}^n_j)|\leqslant\tau k|\bar{u}^n_j|\leqslant\tau k\,\frac{1}{(1-\tau_0 k)}\,\mathrm{e}^{\delta_0 T}\|u^0\|_{L^\infty(\mathbb{Z})}$$

we can write

$$\|u_\tau-\bar{u}_\tau\|_{L^1(K)}\leqslant\mathrm{mes}(K)\tau k\,\frac{1}{(1-\tau_0 k)}\,\mathrm{e}^{\delta_0 T}\|u^0\|_{L^\infty(\mathbb{Z})},$$

then

$$\lim_{\tau\to 0}\|u_\tau-\bar{u}_\tau\|_{L^1(K)}=0$$

and this proves that $u=v$ in $L^1_{\mathrm{loc}}(\mathbb{R}\times]0,T[)$.

Let us now show that u satisfies the entropy condition (9). Scheme (13) is monotone under the CFL condition (14); then using a result of Tadmor [15], there exists a numerical entropy flux G associated with the entropy η such that

$$\eta(u^n_{j+1})-\eta(\bar{u}^n_j)+r[G(\bar{u}^n_j,\bar{u}^n_{j+1})-G(\bar{u}^n_{j-1},\bar{u}^n_j)]\leqslant 0.$$

From the convexity of η, we obtain

$$\eta(u^n_{j+1})-\eta(u^n_j)+r[G(\bar{u}^n_j,\bar{u}^n_{j+1})-G(\bar{u}^n_{j-1},\bar{u}^n_j)]\leqslant\eta'(\bar{u}^n_j)(\bar{u}^n_j-u^n_j)=\tau\eta'(\bar{u}^n_j)q(\bar{u}^n_j)$$

which can be written as

$$B'_1+B'_2\leqslant B'_3.$$

Let ϕ be a positive function of $\mathscr{D}(\mathbb{R}\times]0,T[)$, we note $\phi^n_j=\phi(x_j,t_n)$, and use the following notations: $G_\tau(x,t)=G(\bar{u}^n_j,\bar{u}^n_{j+1})$ with $x_j<x<x_{j+1}$ and $t_n<t\leqslant t_{n+1}$, and $\phi_\tau(x,t)=\phi^n_j$ for $x_{j-1/2}<x<x_{j+1/2}$ and $t_n<t\leqslant t_{n+1}$, and define

$$B_l=\sum_{n=0}^{N}\sum_{j\in\mathbb{Z}}hB'_l\phi^n_j,\quad l=1,2,3.$$

One has

$$B_1+B_2\leqslant B_3.$$

Using a discrete integration by parts we get

$$B_1+B_2=-\sum_{n=0}^{N}\sum_{j\in\mathbb{Z}}\tau h\eta(u^n_{j+1})\,\frac{(\phi^{n+1}_j-\phi^n_j)}{\tau}-\sum_{n=0}^{N}\sum_{j\in\mathbb{Z}}h\tau G(\bar{u}^n_j,\bar{u}^n_{j+1})\,\frac{(-\phi^n_{j+1}-\phi^n_j)}{h},$$

hence

$$B_1+B_2=-\int_{\mathbb{R}}\int_0^T[\eta(u_\tau(x,t))(\phi_\tau(x,t))_t+G_\tau(x,t)(\phi_\tau(x,t))_x]\,\mathrm{d}x\,\mathrm{d}t.$$

Making use of Lebesgue's theorem we prove that

$$\lim_{\tau\to 0}(B_1+B_2) = -\int_{\mathbb{R}}\int_0^T [\eta(u)\phi_t + F(u)\phi_x]\,\mathrm{d}x\,\mathrm{d}t$$

and with the Lipschitz property of q, we obtain

$$\lim_{\tau\to 0} B_3 = \int_{\mathbb{R}}\int_0^T \eta'(u)q(u)\psi\,\mathrm{d}x\,\mathrm{d}t.$$

Then u satisfies the entropy condition. □

4. Numerical experiments

In this section, we present some numerical experiments that demonstrate the performance of the scheme previously introduced. We consider here the shallow water equations, which govern free surface flow of an incompressible fluid,

$$\frac{\partial W}{\partial t} + \frac{\partial F(W)}{\partial x} = S_{0x} - S_{\mathrm{f}x}, \tag{23}$$

$$W = \begin{pmatrix} h \\ hu \end{pmatrix}, \quad F(W) = \begin{pmatrix} hu \\ hu^2 + g\,\dfrac{h^2}{2} \end{pmatrix}, \tag{24}$$

$$S_{0x} = \begin{pmatrix} 0 \\ gh\alpha'(x) \end{pmatrix}, \quad S_{\mathrm{f}x} = \begin{pmatrix} 0 \\ \dfrac{gN^2u|u|}{h^{1/3}} \end{pmatrix}. \tag{25}$$

In the above equations, h and u are water depth and mean flow velocity, respectively, g is the gravity acceleration, α designates the bed variation and N is the Manning coefficient.

In the last few years, many papers have been devoted to the numerical simulation of catastrophic stream following a collapsed dam. As a system of nonlinear hyperbolic partial differential equations, the homogeneous part of the system is responsible for most difficulties found when equations are numerically integrated. This part is computed by the Roe scheme [13] with entropy modification coupled to well-known MUSCL technique introduced by Van Leer [16] to increase precision. A Runge–Kutta order two is used for time integration only to decrease the calculation time. It has been explained and proved by Ambrosi [1] that a simple explicit discretization of source terms in Eq. (23) leads to numerical instabilities. Let us write $q = hu$, as a conservative variable and $(S_{0x} - S_{\mathrm{f}x})_2$ the second components of the source term. The method we present here consists of a splitting procedure. First, one devises an approximate solution to the following ODE:

$$\frac{\partial q}{\partial t} = (S_{0x} - S_{\mathrm{f}x})_2 \tag{26}$$

at point (t_n, x_j) by semi-implicit method, this consists in writing (26) as below:

$$\frac{\tilde{q} - q_j^n}{\Delta t} = gh_j^n S_0 - n^2 g\tilde{q}\,\frac{|u_j^n|}{(h_j^n)^{4/3}}.$$

Table 2
Geometrical and physical data

Designation	Value
Length of channel (L)	122 m
Width of channel (l)	1.22 m
Bed slope (α')	0.005
Kinematic viscosity (v)	0.000000113 m^2/s
Coefficient (C)	8133

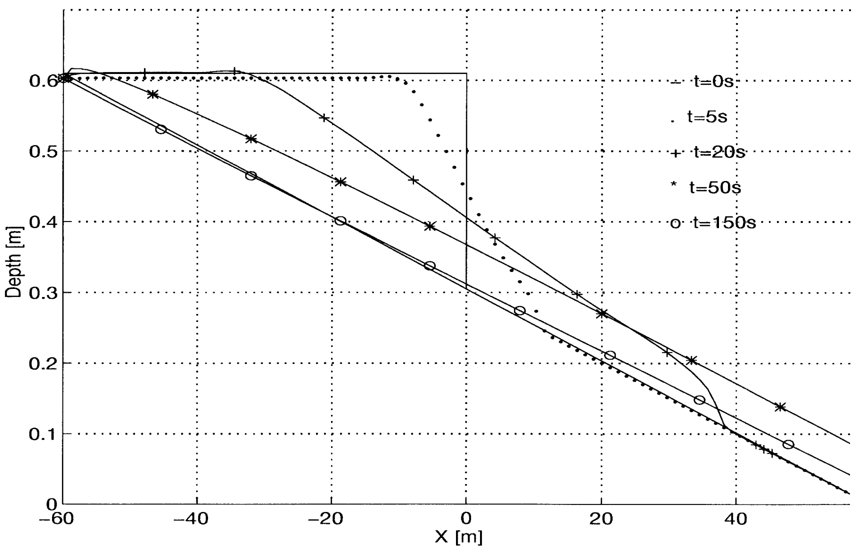

Fig. 2. Water surface profiles for test 1.1 ($\alpha' = 0.005$, $N = 0.009$).

In the second stage, the homogeneous hyperbolic equation is solved with $\tilde{W} = (h^n, \tilde{q})^{\mathrm{T}}$ as initial condition, since in this situation $\tilde{h} = h^n$.

Two experiments carried out at the United States Army Engineer Waterways Experiment Station WES [18] were considered and identified as tests 1.1 and 1.2. These tests 1.1 and 1.2 were performed under dry- and wet-bed conditions downstream of the dam respectively. A model of rectangular channel with the characteristics given in Table 2 were used.

We consider now a Riemann problem:

$$\begin{cases} h(x,0) = h_0 \left(1 + \dfrac{2x}{L}\right) & \text{for } -\dfrac{L}{2} \leqslant x \leqslant 0, \\ h(x,0) = 0 & \text{for } x \geqslant 0. \end{cases}$$

Before suddenly removing the dam the water depth is initially fixed at $h_0 = 0.305\,\mathrm{m}$ while the water is at rest. The boundary conditions were obtained by having water depth equal to zero at downstream and discharge equal to zero at the upstream.

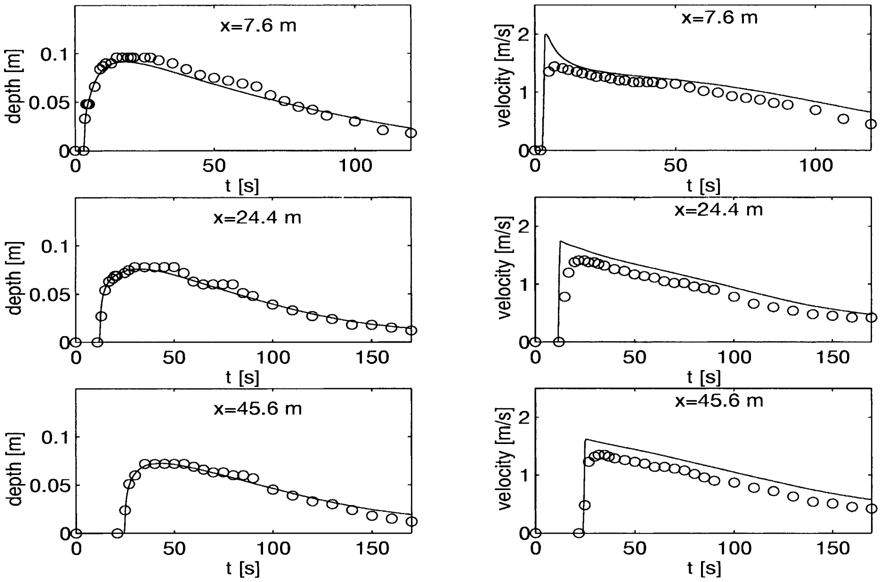

Fig. 3. Depth and velocity hydrographs for experiment 1.1. Circle: measured. Solid: computed.

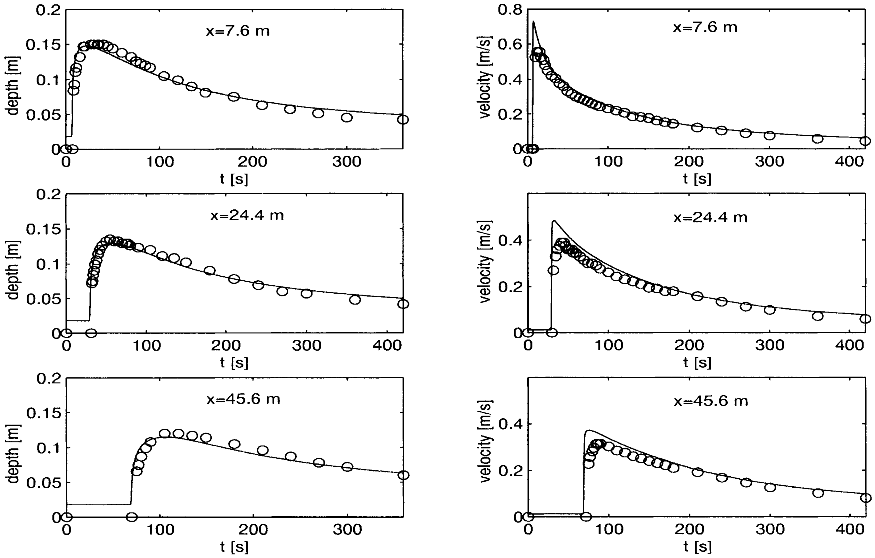

Fig. 4. Depth and velocity hydrographs for experiment 1.2. Circle: measured. Solid: computed.

For test 1.1, the Manning coefficient was experimentally evaluated at 0.009 $s/m^{1/3}$. In Fig. 2, the water surface profiles at 5, 20, 50 and 150 s are reported. Fig. 3 shows the comparison between depth- and velocity-computed hydrographs at $x = 7.6$, 24.4 and 45.6 m and the experimental values at the same abscissae. There is a good agreement between measurements and computation, while the velocities are slightly higher than the experimental ones. We can note good agreement between the observed and computed wave front.

For test 1.2, WES [18] test shows that the Manning coefficient is varying with the flow depth. For this reason, the following relation is considered:

$$N = \sqrt{n_0^2 + \left(\frac{1.486 v C}{8 g \alpha'^{1/2} R^{4/3}}\right)^2},$$

where $n_0 = 0.0354$ is the asymptotical value of Manning and R is the hydraulic radius. The water depth at downstream is equal to 0.0183 m. In Fig. 4, we observe the same behaviour as in test 1.1.

5. Conclusions

The study of splitting scheme has been presented in the scalar case. Because of the effect of source term, the constructed scheme is not TVD, but it has TVB property. Owing to the implicit account of the source term, the numerical solution converges towards the entropy solution. The resolution of one-dimensional shallow water is proposed using splitting method. The proposed scheme gives satisfactory results. The forthcoming work is going towards complicated source terms and generalizes to systems in two space dimensions with weak condition on q.

References

[1] D. Ambrosi, Approximation of shallow water equations by Riemann Solvers, Internat. J. Numer. Methods Fluids 20 (1995) 157–168.

[2] F. Benkhaldoun, A. Chalabi, Characteristic based scheme for conservation laws with source terms, Université Paul Sabatier, Mathématiques pour l'Industrie et la Physique, Rapport interne no. 95.13, 1997.

[3] A. Chalabi, Stable upwind schemes for hyperbolic conservation laws with source terms, IMA J. Numer. Anal. 12 (1992) 217–241.

[4] I. Elmahi, F. Benkhladoun, Schémas volumes finis pour la convection et la diffusion dans un problème d'inflammation de goutte en maillages non structurés adaptatifs, Neuvième Séminaire sur les Ecoulements de Fluides Compressibles, Institut des Sciences et Techniques Nucléaires, CEA Saclay1, 1997.

[5] P. Glaister, Second order difference schemes for hyperbolic conservation laws with source terms, University of Reading, Department of Mathematics, Numerical Analysis, Report No. 6/87, 1987.

[6] S. Jin, Runge–Kutta methods for hyperbolic conservation laws with stiff relaxation terms, J. Comput. Phys. 122 (1995) 51–67.

[7] S.N. Kruzkov, First order quasi-linear equations in several independent variables, Math. U.S.S.R Sb. 10 (1970) 217–243.

[8] R.J. Leveque, H.C. Yee, A study of numerical methods for hyperbolic conservation laws with stiff source terms, J. Comput. Phys. 86 (1990) 187–210.

[9] T.P. Liu, Hyperbolic conservation laws with relaxation, Comm. Math. Phys. 108 (1987) 153–175.

[10] F. Macchione, M.A. Morelli, Application of TVD-Lax-Wendroff schemes to Dam Break wave propagation, Proceedings Third International Conference of CMWR, Beyrouth, Lebanon, 1995.

[11] L.A. Monthé, Etude numérique des équations aux dérivées partielles hyperboliques. Application au cas de Saint-Venant, Thèse de doctorat, LMI-INSA de Rouen, 1997.

[12] P.A. Raviart, E. Godlewski, Hyperbolic Systems of Conservation Laws, Ellipses, 1991.

[13] P.L. Roe, Approximate Riemann Solvers, Parameter vectors, and difference scheme, J. Comput. Phys. 43 (1981) 357–372.

[14] H.J. Schroll, W. Winter, Finite difference schemes for conservation laws with source terms, IMA J. Numer. Anal. 16 (1996) 201–215.
[15] E. Tadmor, Numerical viscosity and the entropy condition for the conservative difference schemes, Math. Comp. 43 (1984) 369–382.
[16] B. Van Leer, Towards the ultimate conservative difference scheme V. A second order sequel to Godunov's method, J. Comput. Phys. 23 (1979) 101–136.
[17] J.P. Vila, On the existence and the smoothness of solutions for fluid flow equations with friction and slope source term, Internal Report, Ecole Polytechnique, No. 133, 1985.
[18] Waterways Experiment Station, United States Army Corps of Engineers, Floods resulting from suddenly breached dams, Miscellaneous paper No. 2-374, Vicksburg, Mississipi, Report 1: conditions of minimum resistance, 1960, Report 2: conditions of high resistance, 1961.